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Laser-target technique for determination of elastic constants of glass over a wide temperature range

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Optical methods have been developed for high temperature application in the determination of the basic quantities, E, Young's modulus and ν, Poisson's ratio. The methods are simple and the techniques are inexpensive. The methods presented have been tried on commercially available soda-lime–silica glass and consistent results were obtained.

1. Introduction

The anticlastic effect in flat beams was observed by Galileo in early seventeenth century. This effect has been analysed by Saint Venant in Navier [1], and Lamb [2]. Searl [3] developed an optical method in 1908 to observe this effect. Ashwell and Greenwood [4] partially verified Lamb's theory in 1950. In 1964, Bellow *et al.* [5] used strain gauges to verify Lamb's theory quite extensively.

Lamb's theory shows that if $(b^2/R_1h) < 1.6$, the anticlastic deformation is described by an arc of radius R_1/ν, where ν denotes Poisson's ratio, R_1 the longitudinal radius of curvature, b the width and h the thickness of the rectangular plate subjected to deformation. The theory is developed, of course, within the range of linear elasticity.

This effect has been used extensively to determine E and ν for glass by the Newton's ring method. The present work is intended to improve the classical Newton's ring method for room temperature and also to develop new techniques for the determination of these basic physical quantities at elevated temperature.

2. Newton's ring method

The classical method of Newton's ring uses the optical interference pattern produced between an optical flat and a deformed specimen as shown in Fig. 1a. If r is the distance to a fringe of order n, the radius of curvature R will be given by

$$R = \frac{r^2}{n\lambda} \qquad (1)$$

where λ is the wavelength of monochromatic light used. The longitudinal and the transverse radius of curvature, R_1 and R_b, respectively, are evaluated from the fringes in the corresponding directions. Young's modulus is then given by

$$E = \frac{6PlR_1}{bh^3} \qquad (2)$$

and Poisson's ratio by

$$\nu = \frac{R_1}{R_b}.$$

provided $b^2/R_1h < 1.6$. Poisson's ratio can also be determined using the relation,

$$\nu = \frac{1}{\tan\alpha} \qquad (3)$$

where α is the angle between the asymptote of the hyperbolic fringes and the longitudinal axis as shown in Fig. 1a.

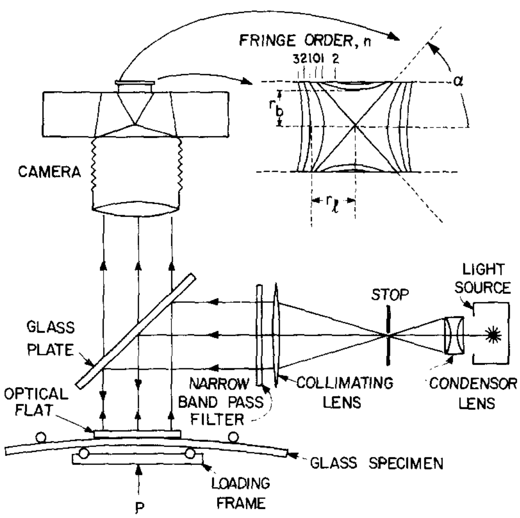
The method is subject to serious error if the specimen surface is not optically flat. In the case of transparent material available in the form of large plates, the plates are scanned in a well collimated large-field laser beam at a small angle of incidence. The degree of flatness is then determined by observing the interference pattern in the reflected light beam. An example of these fringes is shown in Fig. 3b. The sections of the plates which do not produce any fringes in the reflected beam are used for making the specimen. This method of selecting an optically flat specimen has been found to be superior to that of using an optical flat. The latter technique, even though quite common, is time consuming and endangers the surface quality of the optical flat.

An experimental result on commercial plate-glass at room temperature is shown in Fig. 1b. The holographic technique [6, 7] has been applied to this problem also which eliminates the use of optical flat.

3. Collimated laser-beam technique

At elevated temperatures, neither the Newton's ring method nor the holographic technique can be used because the use of optical flat in the former method and the turbulence in the hot air in the latter method cause major problems.

A well-collimated He–Ne Laser beam of circular cross-sectional diameter, D, is reflected by a glass lath specimen located in a loading frame as shown in Fig. 2. The reflected beam travels a distance r to the film plane of a camera. In the absence of any load, the light beam is reflected unaltered; under the influence of a load, the cross-section of the beam takes an elliptical shape.



$$R_l = \frac{(r_l)^2}{n\lambda} \quad R_b = \frac{(r_b)^2}{n\lambda} \quad \nu = \frac{R_l}{R_b} \text{ ALSO } \nu = \frac{1}{\tan^2 \alpha}$$

(a)

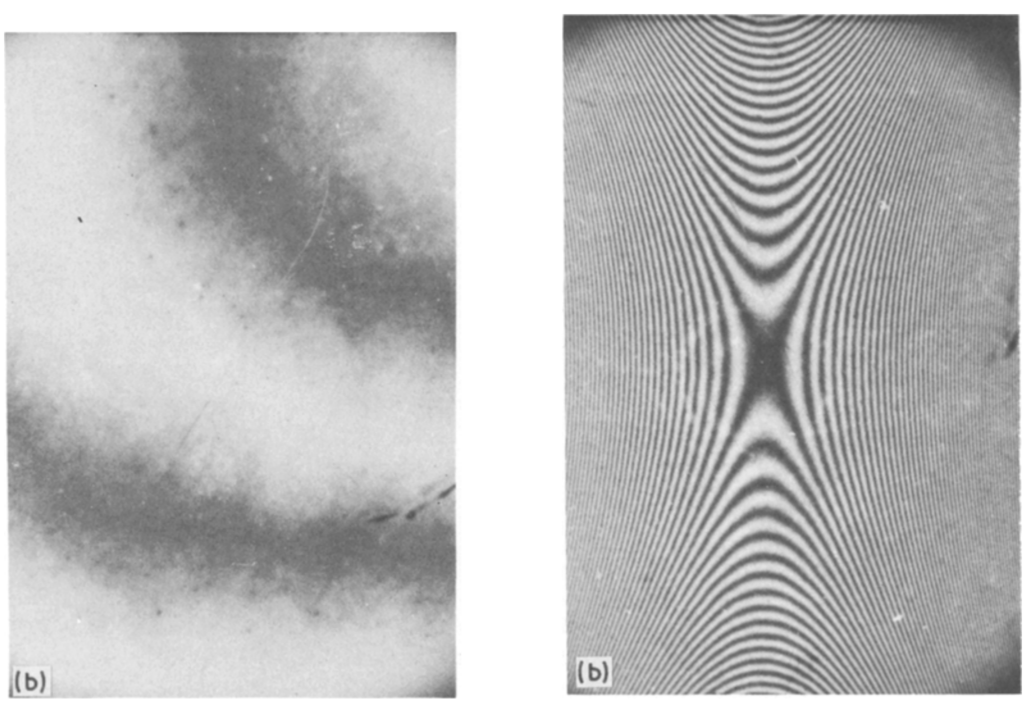


Figure 1(a) Schematic diagram of experimental arrangement for the determination of Young's modulus, E, and Poisson's ratio, ν, by measuring the anticlastic effect of glass plate in pure bending using Newton's ring method. (b) Photographic example of the Newton's rings. (Left, unloaded; right, loaded.)

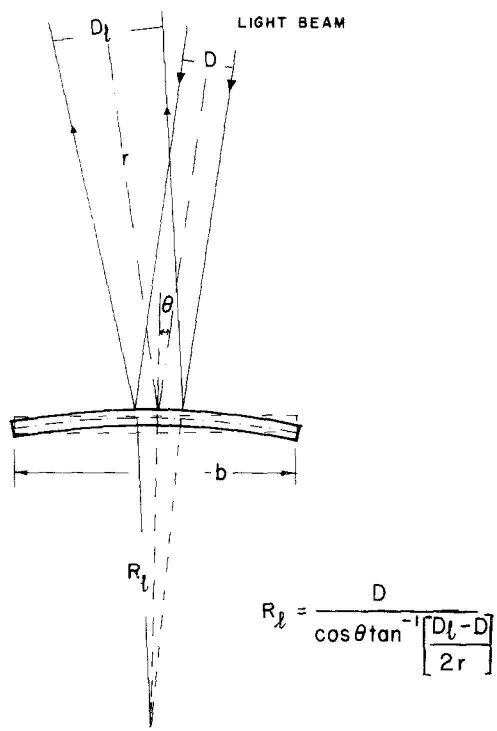


Figure 2 Principle of collimated laser beam technique for determination of E and ν.

If D_l and D_b are respectively the longitudinal (major) and the transverse (minor) dimensions of the deformed cross-section of the light beam, then for $\theta \approx 0$ and $r \gg D$,

$$R_l = \frac{2Dr}{D_l - D} \tag{4a}$$

and

$$R_b = \frac{2Dr}{\cos\theta\,(D_b - D)} \tag{4b}$$

and for $b^2/R_l h < 1.6$,

$$\nu = \frac{(D_b - D)\cos\theta}{D_l - D} \tag{5a}$$

and

$$E = \frac{12\,PlDr}{bh^3\,(D_l - D)}\ . \tag{5b}$$

The optical and loading systems are shown schematically in Fig. 3a. A typical experimental result, using $r = 4$ m and $\theta = 3°$, is demonstrated in Fig. 3b. The cross-section of the laser beam was found not to be perfectly circular. In this case, the value of D was modified by using the corresponding characteristic dimensions of the beam. Fig. 3a also shows a target in the form of a zone plate printed on a clear plastic substrate and a target lens. In the present experiments, the target and the target lens were not used. The application of these two elements of the optical system will be clarified in the following section.

4. Laser-target technique

There are two different ways a target in a collimated laser-beam can be used for the determination of the elastic response of a deformed rectangular beam.

4.1. Method 1

With the specimen unloaded, a target lens of long focal length (Fig. 3a) is adjusted to have the image of the target on the film plane of a camera (without any lens) equipped with a focal plane shutter. The position of the camera is chosen to accommodate the image fully. The image of the target is deformed, as shown in Fig. 3c, when a given load is applied to the specimen. A zone plate drawn on a transparent plastic plate is used in this case as the target object. If the camera is positioned far away from the specimen, then the optical situation in the present experiment is analogous to the previous method. If D is the diameter of a ring before loading and D_l, D_b are, respectively, the longitudinal and transverse dimensions of the deformed image, then both E and ν can be determined using Equation 5. The camera is synchronized to record the deformed image within a fraction of a second after the application of the load. The measured values therefore give, very precisely, the elastic response even at elevated temperature where the material behaves viscoelastically.

In Fig. 3b and c, the background interference fringes result from the reflection from both surfaces of the specimen. This can be partially eliminated by selecting a specimen with optically flat and parallel surfaces. The interference fringes can be eliminated completely by the deposition of a thin film of gold on the front surface of the specimen. This also increases the reflectivity of the specimen surface. The gold-coated specimen could also be used at high temperature. An example of using such a specimen is shown in Fig. 3d.

4.2. Method 2

With the specimen unloaded, a target lens of long focal length is adjusted to give an image of the

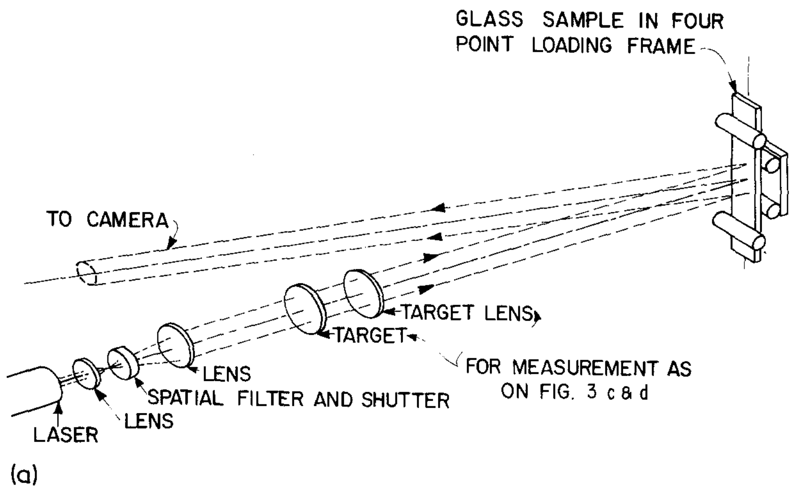


Figure 3(a) Schematic diagram of the experimental system for collimated laser beam and laser target technique. (b) Photographic recording of the distortion of the collimated laser beam by the anticlastic surface. (Left, not loaded; right, loaded.) (c) Photographic recording of the distortion of the target image due to anticlastic effect. The interference fringes in the background result from the reflection from both surfaces of the plate. (Left, not loaded; right, loaded.) (d) Elimination of the background fringes in the laser target system by vacuum deposition of a thin film of aluminium or gold on the front surface of the specimen. (left, before loading; right, after loading.)

target at infinity. The anticlastic surface of a loaded specimen acts like a mirror, having two different curvatures orthogonal to each other. With the arrangement of the specimen as shown in Fig. 3a, a real image of part of the target corresponding to the transverse direction, is produced at half the distance of the radius of curvature, whereas the section corresponding to the longitudinal direction produces a virtual image. The sections of the target lying on the vertical line come to sharp focus at the focal distance corresponding to the transverse direction. The sections along the horizontal line, however, exhibit a system of satellite fringes which could be seen only on close examination. These fringes appear in other directions also if the screen or the camera is moved away from the focal plane. This particular characteristic of the image can be used to determine accurately the position of the focal plane corresponding to the transverse direction and hence the radius of curvature, R_b. To determine R_l separately, an additional convex lens of focal length f is necessary to introduce in the reflected beam to bring the horizontal sections of the target into focus. The position of this lens and the camera body is adjusted until the horizontal sections of the image come to sharp focus. If r_s and r_i are the distances of the specimen surface and the film plane from the principal plane of the additional lens, then

$$R_l = 2\left(\frac{fr_i}{r_i - f} - r_s\right). \qquad (6)$$

Once R_l and R_b are known for a given load, both E and ν can be estimated using Equation 2, provided $b^2/R_l h < 1.6$.

This method is time consuming. The experimental time, however, can be reduced to about 10 min with practice.

5. Discussion

The applicability of these methods is determined by the measuring time. The elastic response, by definition, is attributed to the instantaneous deformation after the application of the load. The proximity of the measured values of E and ν to their true value depends entirely on the amount of anelastic deformation that occurs during the measuring time.

All the above-mentioned methods have been tried on specimens prepared from commercial plate-glass of the soda-lime–silica type. This glass is known to be perfectly elastic, under moderate stress, at room temperature. All the methods gave, within experimental error, the same results for both E (7.6×10^5 kg cm^{-2}) and ν (0.201) for the same glass specimen at room temperature. The modulus of elasticity, E, determined from the measurement of the deflection of the beam agreed,

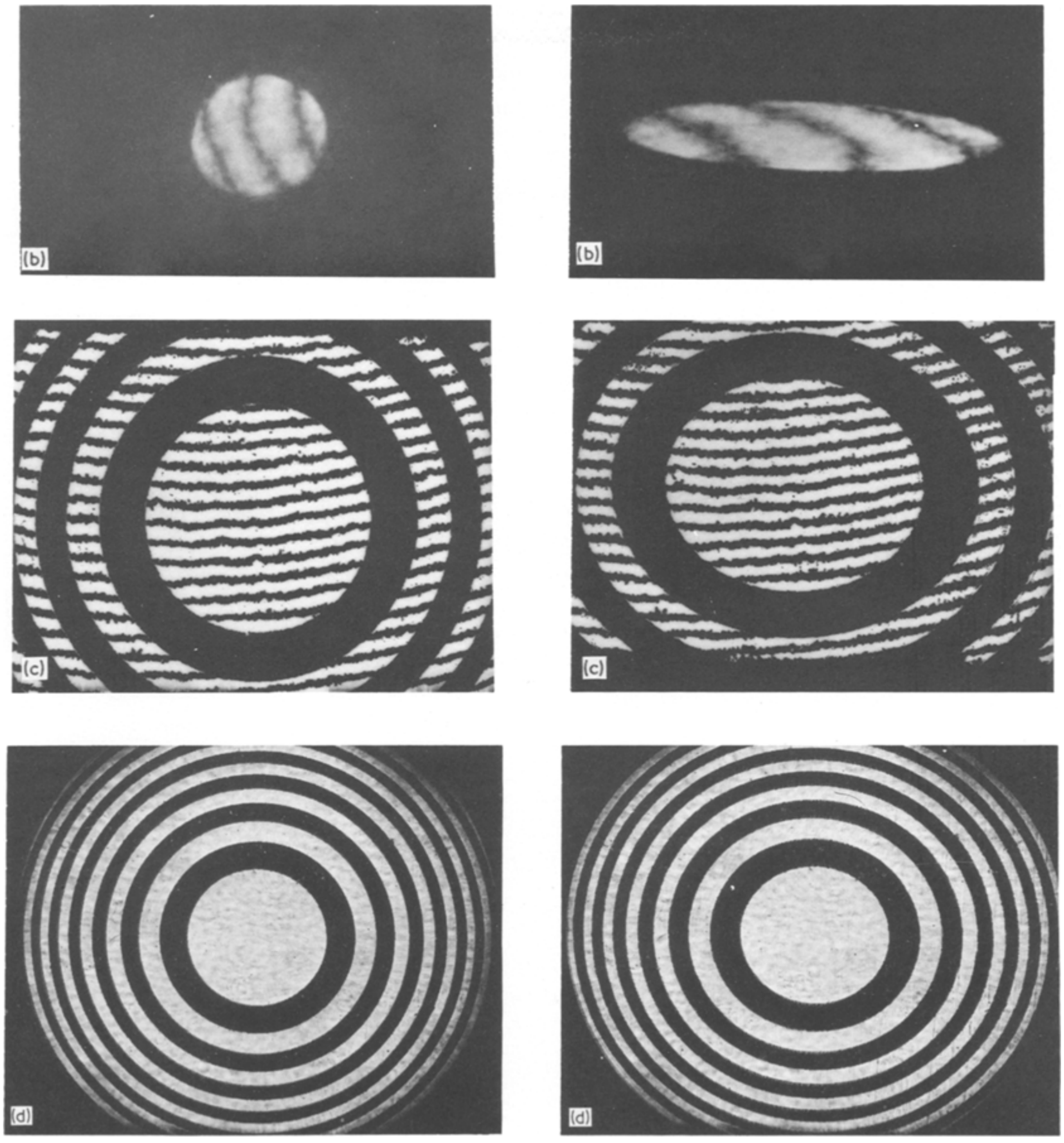


Figure 3 continued.

within the error of measurement, with the values determined by the optical method.

The laser-target technique method 2, having a measuring time of about 10 min, imposes a limit to the high temperature application. It is estimated that the method can be applied to soda-lime–silica glass up to about 350° C without introducing any significant error due to viscoelastic flow. The error will become progressively larger with increasing temperature. It is estimated [8] that even a 5 sec modulus, i.e. measured 5 sec after the application of the load, introduces an error of 13% at 550° C, an error of 35% at 575° C and an error of 70% at 600° C. The target technique method 1, and the collimated laser-beam technique can be used safely up to about 580° C for plate glass since the measuring time in both cases can be reduced to a fraction of a second.

Glass is not a Newtonian fluid in the strict sense at high temperature [8, 9]. The response of glass, however, under the influence of a constant stress, approaches that of a Newtonian fluid as a limiting case. The applicability of the condition of incompressibility is, therefore, limited to the steady state deformation. This has been observed directly by the application of the optical techniques. The anticlastic surface of a rectangular plate in bending was observed by the laser target technique method 1 during the steady state viscous flow at 600° C. It was observed that the lateral radius of curvature maintained a constant ratio of 2:1 with the longitudinal radius of curvature. This has also been observed experimentally on soda-lime glass in creep experiments. A glass specimen was allowed to deform by viscous flow to a strain of about 30% under a uniaxial tensile stress at 625° C. The decrease in the cross-sectional area agreed with the value expected assuming incompressibility [8].

6. Conclusion

A few simple optical techniques have been developed for the determination of Young's modulus and Poisson's ratio. The methods are particularly applicable to glass or any other reflective material available in the form of flat plates. The merit of the collimated laser-beam technique and the laser-target technique method 1 lies in their applicability at high temperatures.

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